

Accuracy and Efficiency Considerations in the Solution of Extremely Large Electromagnetics Problems

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Abstract—Fast and accurate solutions of extremely large electromagnetics problems are considered. Surface formulations of large-scale objects lead to dense matrix equations involving millions of unknowns. Thanks to the recent developments in parallel algorithms and high-performance computers, these problems can easily be solved with unprecedented levels of accuracy and detail. For example, using a parallel implementation of the multilevel fast multipole algorithm (MLFMA), we are able to solve electromagnetics problems discretized with hundreds of millions of unknowns. Unfortunately, as the problem size grows, it becomes difficult to assess the accuracy and efficiency of the solutions, especially for the comparison of different implementations. This paper presents our efforts on the solution of extremely large electromagnetics problems with the emphasis on the accuracy and efficiency. We present a list of benchmark problems, which can be used to compare different implementations for large-scale problems.

I. INTRODUCTION

Real-life electromagnetics problems often involve very large objects with respect to wavelength. Accurate formulations of these problems with the surface integral equations lead to dense matrix equations involving millions of unknowns. Recently, there has been many efforts to increase the problem size from millions to hundreds of millions using high-performance computing techniques on parallel computers [1]–[14]. Specifically, parallelization of the fast algorithms, such as the multilevel fast multipole algorithm (MLFMA) [15], has enabled the solution of extremely large electromagnetics problems, discretized as many as one billion unknowns [16]. At the same time, with the rapid increase in the number of unknowns, it becomes crucial but more difficult to assess the accuracy and efficiency of the algorithms and their parallel implementations [17].

This paper presents our efforts for rigorous solutions of extremely large electromagnetics problems. We discuss the assessment of the accuracy and efficiency of parallel algorithms and their implementations. Based on these discussions, we present a set of benchmark problems involving canonical objects to test and compare different solvers for large-scale

problems. We also present the solution of these benchmark problems using a sophisticated parallel implementation of MLFMA. Details of this implementation can be found in [9] and [14].

II. EFFICIENCY CONSIDERATIONS

Measuring the efficiency of a parallel solution is not trivial. Parallelization speedup and efficiency results, which are often used to demonstrate the effectiveness of the parallel implementations, can be misleading [9]. A major problem is that the parallelization speedup is usually measured as the reduction in the processing time with respect to a single-processor solution, but it does not give any information on the processing time on a single processor, i.e., the efficiency of the algorithm itself. Specifically, a very slow algorithm can be “embarrassingly” parallelizable leading to very high parallelization speedup and efficiency, but this does not mean that the implementation and solutions are efficient. Unfortunately, a very common practice in the literature is to increase the parallelization speedup and efficiency by increasing the processing time, e.g., by performing some of the computations on the fly and parallelizing them very efficiently. Obviously, these implementations may not be as efficient as claimed and the related efficiency results are often exaggerated.

Another disadvantage of using the parallelization speedup and efficiency is their complicated dependence to the parallel computer and architecture. For example, using slower processors without changing the communication network may increase the parallelization efficiency. This is because the computations, which take longer on the slower processors, dominate the communications. Having relatively short communication time in the overall processing time translates into higher parallelization speedup and efficiency. But, again, the actual efficiency of the solution can be very low due to the slower processors.

Considering the disadvantages of using the parallelization speedup and efficiency, the processing time itself can be used

to measure the actual efficiency of the parallel implementations. The processing time also depends on the parallel computer. But, computer specifications and solution parameters, such as the number of processors, distribution of processes, processor models, clock rates, and the network speed can be provided along with the processing time. This may lead to more fair comparisons of the parallel implementations. Solutions of multiple problems with different sizes can increase the reliability of the comparisons.

III. ACCURACY CONSIDERATIONS

Accuracy is easier to assess but often discarded in large-scale computations [17]. There are many error sources in numerical solutions, such as the discretization of the geometry and equations, numerical integration, factorization, diagonalization, interpolation, and iterative convergence. All these error sources must be suppressed to obtain accurate results. Accuracy of solutions must accompany the efficiency results. For example, reducing the accuracy may reduce the processing time and increase the parallelization efficiency. Unfortunately, decreasing the number of integration points (in numerical integrations), decreasing the number of harmonics (in factorizations), decreasing the number of samples (in interpolations), and increasing the target residual error (in iterative solutions) are commonly practiced in the literature. Some of these relaxations may have significant effects in the final solutions. Hence, without indicating the accuracy, the efficiency results can be again exaggerated and misleading to assess the effectiveness of the implementations.

IV. BENCHMARK PROBLEMS

In order to facilitate the assessment of parallel implementations and for their fair comparisons, we determined a set of benchmark problems involving two canonical objects, namely, the sphere and NASA Almond [19]. The sphere has a radius of 0.3 m and it is investigated at 11 different frequencies from 20 GHz to 340 GHz. The NASA Almond has a size of 0.252474 m and it is investigated at 5 different frequencies from 112.5 GHz to 1.8 THz. Both objects are illuminated by plane waves. For the NASA Almond, two different illuminations are considered; head-on illumination and 30° illumination. In both cases, the electric field is polarized horizontally. We provide the reference solutions of these problems in an interactive web site at www.cem.bilkent.edu.tr/benchmark. Computational values for the far-zone electromagnetic fields can be uploaded in this site. For the sphere, the uploaded results are compared to the analytical Mie-series solutions (obtained with 10^{-6} error). For the NASA Almond, comparisons are made against our numerical solutions (obtained with 10^{-2} error). In all cases, relative errors are calculated to assess the accuracy of the numerical solutions. In the next section, we present examples to the solutions of the benchmark problems performed by our parallel implementation of MLFMA.

TABLE I
SOLUTIONS OF SCATTERING PROBLEMS INVOLVING A SPHERE OF
RADIUS 0.3 M AT VARIOUS FREQUENCIES

Diameter	Unknowns	Iterations (10^{-3} Residual)	Total Time (minutes)	Error in RCS
40.0λ	1,462,854	21	4	0.0092
80.1λ	5,851,416	27	16	0.0097
160.1λ	23,405,664	33	61	0.0099
192.1λ	33,791,232	39	107	0.0098
240.2λ	53,112,384	44	183	0.0099
320.2λ	93,622,656	47	333	0.0104
360.3λ	135,164,928	47	471	0.0093
420.3λ	204,823,296	50	647	0.0071
520.4λ	307,531,008	55	1080	0.0089
560.4λ	374,490,624	58	1430	0.0085
680.5λ	540,659,712	65	3632*	0.0084

* Parallelized into 64 processes.

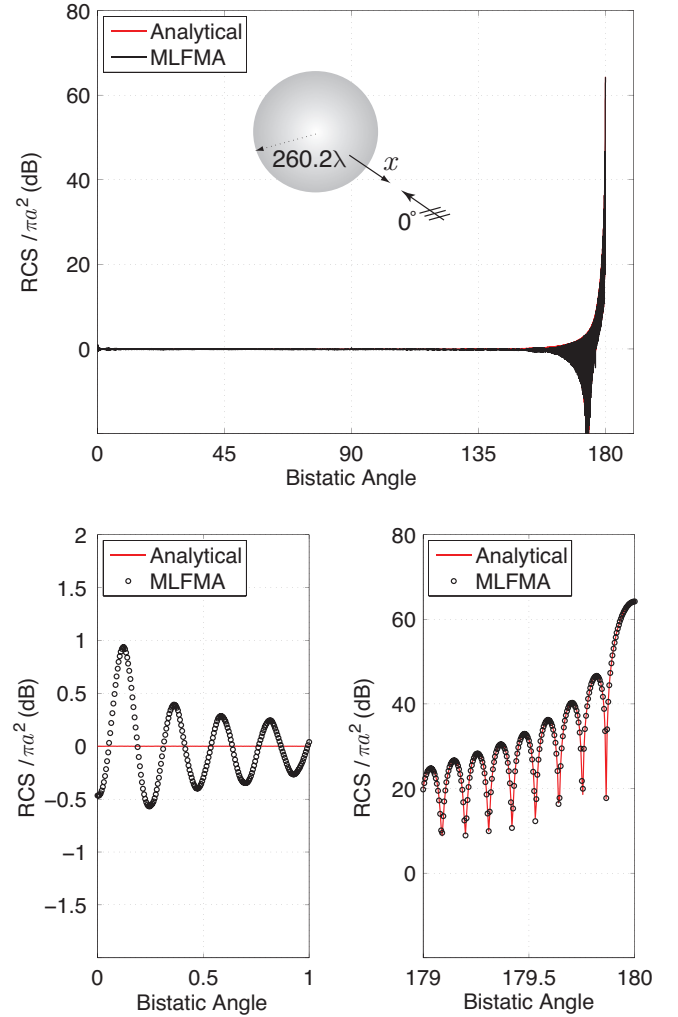


Fig. 1. Normalized bistatic RCS ($RCS/\pi a^2$) of a sphere with a radius of 340λ from 0° to 180° , where 0° and 180° correspond to the back-scattering and forward-scattering directions, respectively. RCS values are zoomed around the back-scattering and forward-scattering directions in separate plots.

V. NUMERICAL RESULTS

For numerical solutions, we use a parallel implementation of MLFMA based on the hierarchical partitioning strategy [9].

All problems are formulated with the combined-field integral equation [20] using 0.5 combination parameter and discretized with the Rao-Wilton-Glisson functions [21] on $\lambda/10$ triangles, where λ is the wavelength. Matrix elements are calculated with maximum 1% error. Interpolations and antepolations are performed locally by using 6×6 stencils. Iterative solutions are performed by using the biconjugate-gradient-stabilized (BiCGStab) algorithm [22] and iterations are carried out until the residual error is reduced to below 10^{-3} . With these parameters, the target error in the far-zone electromagnetic fields is 1%.

Table I presents the solution of the sphere problems on a cluster of Intel Xeon Nehalem-EX L7555 processors with 1.87 GHz clock rate. All solutions are parallelized into 128 processes, except the solution of the largest problem that is parallelized into 64 processes. Processes are distributed among 16 computing nodes. The diameter of the sphere changes approximately from 40λ to 540λ , whereas the corresponding number of unknowns changes from 1,462,854 to 540,659,712. Table I lists the number of iterations, the total computing time in minutes, and the error in the far-zone electromagnetic fields. Note that the smallest and largest problems are solved in about 4 minutes and 60 hours, respectively. The error in the far-zone fields is generally less than 1%, confirming that the desired accuracy is satisfied for these large-scale problems. Fig. 1 depicts the bistatic radar cross section (RCS) of the sphere at 340 GHz. It can be observed that the computational values agree very well with the analytical results obtained by using a Mie-series solution.

Table II presents the solution of the NASA Almond problems again on a cluster of Intel Xeon Nehalem-EX L7555 processors with 1.87 GHz clock rate. All solutions are parallelized into 64 processes distributed among 16 computing nodes. The size of the NASA Almond changes approximately from 94.7λ to 1515.3λ . The smallest and largest problems are discretized with 2,157,462 and 552,310,272 unknowns, respectively. Table II lists the number of iterations and the total computing time for two different illumination angles. Fig. 2 depicts the bistatic RCS of the NASA Almond at 1.8 THz when it is illuminated by a plane wave propagating towards its nose (head-on illumination). Since the RCS values are obtained with maximum 1% error, Fig. 2 can be used as a reference for high-frequency techniques.

In addition to the canonical problems involving the sphere and NASA Almond, we also employ our parallel implementation to solve more complicated problems. As an example, Fig. 3 presents the solution of a scattering problems involving the stealth airborne target Flamme [23]. The scaled size of the target is 0.6 m and it is investigated at 820 GHz, i.e., when its size is approximately 1641.1λ . Fig. 3 depicts the bistatic RCS of the target for the head-on illumination. As opposed to the RCS of the NASA Almond, the RCS of the Flamme exhibits specular reflections and the cross-polar component is significantly large.

TABLE II
SOLUTIONS OF SCATTERING PROBLEMS INVOLVING THE NASA ALMOND
AT VARIOUS FREQUENCIES

Size	Unknowns	Iterations (10^{-3} Residual)		Total Time (minutes)	
		0°	30°	0°	30°
94.7 λ	2,157,462	27	26	8	8
189.4 λ	8,629,848	31	27	33	31
378.8 λ	34,519,392	40	32	148	134
757.7 λ	138,077,568	47	50	634	654
1515.3 λ	552,310,272	68	70	3269	3396

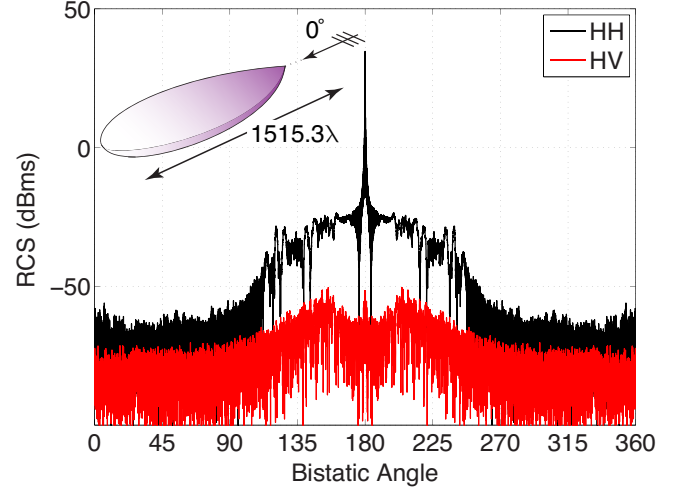


Fig. 2. Co-polar (HH) and cross-polar (HV) bistatic RCS (dBms) of the NASA Almond at 1.8 THz. The target is illuminated by a plane wave propagating towards its nose with the electric field polarized horizontally.

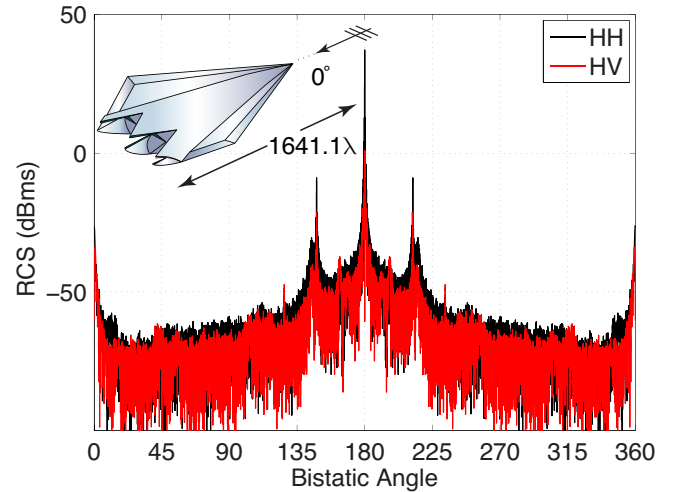


Fig. 3. Co-polar (HH) and cross-polar (HV) bistatic RCS (dBms) of the Flamme at 820 GHz. The target is illuminated by a plane wave propagating towards its nose with the electric field polarized horizontally.

VI. CONCLUSIONS

It is extremely important to precisely assess the accuracy and efficiency of the parallel implementations for large-scale electromagnetics problems. In this study, we present a set of benchmark problems, which can be used to compare

different implementations. We also present rigorous solutions of these problems discretized with millions of unknowns using a parallel implementation of MLFMA. Reference solutions are also available in an interactive web site at www.cem.bilkent.edu.tr/benchmark.

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